Modified gravity models in cosmology

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KIAS-NCTS workshop

High1-2014, February 10, 2013

[with profs. Mukohyama, Tanaka, Tsujikawa]

Introduction

- Einstein theory: great, beautiful achievement
- It sets a high energy scale (M_P), small scales (l_P)
- At low energies? At very large scales?
- What is the connection between small scales and larges scales?

Revolution

- 2011 Nobel Prize: discovery of acceleration at large scales
- What drives it accounts for 68% of the total matter distribution
- What is it?

Cosmological constant

- A cosmological constant consistent with data
- Simplest generalization of GR

$$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2}\left(R - 2\Lambda\right)\right] + S_m$$

- Fundamental constant: prediction from EFT/QFT?

Problem(s)

- What is the phenomenological picture of gravity?
- Can it be interpreted in terms of HEP?
- Vacuum Minkowski solution is lost
- How to connect HEP (very small scale, Minkowski based) to LEP (very large scale, curved background)?
- Up to now, no solution to this problem(s)

Dark Energy/Modifications of Gravity

- Start to find a dynamical mechanism
- Many possibilities: covariance symmetry
- Pure phenomenological approach
  - Set/constrain a model
  - Where does it come from?

DE as a scalar field

- Minimally coupled scalar field, quintessence

$$S=\int d^4x\sqrt{-g}\left[\frac{M_P^2 R}{2}-\frac{1}{2}(\partial\varphi)^2-V(\varphi)\right]+S_m$$

- Mass of quintessence field, $m_\varphi \sim 10^{-33}\,eV$
- In general difficult to accomodate in particle physics
  [Frieman et al. 1995; Hall et al. 2005]

Classification of quintessence

- Classified according to evolution [Caldwell, Linder 2005]
- Equation of state parameter $w = \frac{p_\varphi}{\rho_\varphi} = w(\varphi(z))$
- In FLRW universe, $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad H = \dot{a}/a$

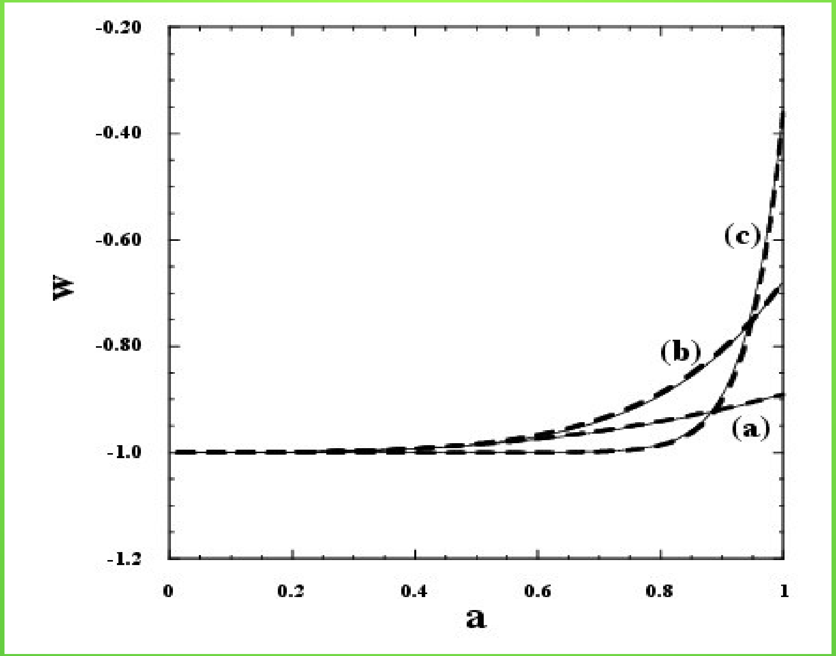
$$3 M_P^2 H^2 = \rho_\varphi + \rho_m ,$$
$$\ddot{\varphi} + 3 H \dot{\varphi} + V_{,\varphi} = 0 ,$$
$$\dot{\rho}_m + 3 H (\rho_m + p_m) = 0 ,$$
$$\rho_\varphi \equiv \frac{1}{2}\dot{\varphi}^2 + V ; \quad \rho_\varphi - p_\varphi = 2V .$$

Constraints from cosmological data

- CMB data (high redshift constraints)
- BAO (low redshift data)
- Supernova Type IA (low redshifts)
- Structure formation, lensing...
- Gravitational waves?

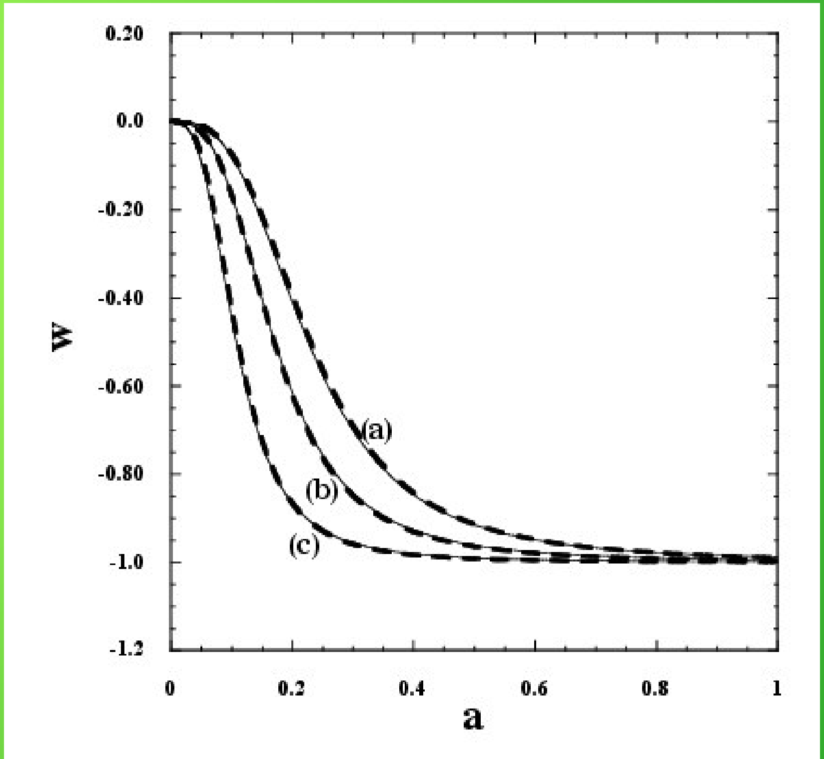
Quintessence: thawing models

- Thawing models: frozen at early times
- Typical example $V = \mu^4 [1 + \cos(\varphi / f)]$



Quintessence: freezing models

- Freezing models: frozen at late times.
- *Scaling freezing*: $w \rightarrow 0 \rightarrow -1$
- Typical example: $V = \mu\left(e^{\lambda_1 \varphi} + \lambda_3 e^{\lambda_2 \varphi}\right)$



Alternatives to quintessence & CC

- What if DE is a change of gravity at large scales?
- Modifications of gravity
- The Einstein-Hilbert action is different
- Extra dimensions
- …

f(R) gravity

[Capozziello: IJMP 2002; ADF, Tsujikawa: LRR 2010]

- Phenomenological model $\mathscr{L}_{GR} = f(R)$
- Working toy models appeared: $f' > 0,\ f'' > 0,\ f(0) = 0$

[Hu et al 2007] $$\frac{f}{M_P^2} = R - \mu R_c [(R/R_c)^{2n} / (1 + (R/R_c)^{2n})]$$

[Starobinsky 2007] $$\frac{f}{M_P^2} = R - \mu R_c [1 - (1 + R^2/R_c^2)^n]$$

[Tsujikawa 2008] $$\frac{f}{M_P^2} = R - \mu \tanh(R/R_c)$$

Scalar-tensor theory approach

- Equivalent to the theory described by

$$S=\int d^4x\sqrt{-g}\left[f(\varphi)+(R-\varphi)f_{,\varphi}\right]$$

- Important mapping: f(R) is a scalar tensor theory
- 1 extra scalar field
- Non-minimally coupled to gravity
- Conformal transformation to Einstein Gravity

f(R) Inflation

[Starobinsky '82]

- First ever model for inflation $f(R) = M_P^2 [R + R^2/(6M^2)]$
- Still compatible with latest Planck data
- $n_R - 1 \simeq 0.964\,, \quad r \simeq 4 \times 10^{-3}$
- In particular, order ε non-gaussianities

Chameleon mechanism in f(R)

- Lagrangian built such that $\lim_{R \to \infty} f(R) = M_P^2 R$
- Then for large R, $R \gg H_0^2$, theory reduces to GR
- But for large R, then $R \approx -T^{\mu}{}_{\mu} / M_P^2$
- In the presence of matter, then theory reduces to GR
- Chameleon mechanism

Solar system constraints for f(R)

- Outside star, GR models vacuum
- But in vacuum R = 0, and we would not have chameleon
- However, in reality, outside the star there is still matter
- This matter, though low-density, is enough to provide enough Chameleon to evade solar system constraints
- Semi-analytic and numerical work requires n > 0.9
  [Capozziello, Tsujikawa: PRD 2008]

Chameleon: not always the case

- Other theories of gravity might not need Chameleon
- Gauss-Bonnet gravity: $\mathscr{L}_{GR} = M_P^2 R + f(G),$ [Nojiri et al 2005]
  - On the solar system, even vacuum GR, $G = 12 r_s^2 / r^6 \gg H_0^4$
  - Do not need outside-matter's Chameleon
  - This theory has unstable FLRW background [ADF et al 2011]
- Other theories, other possibilities

Perturbation evolution on FLRW for f(R)

- At large scales deviation from GR becomes evident
- On studying the growth index $\frac{\dot{\delta}_m}{H\,\delta_m} = \Omega_m^{\gamma}$ [Starobinsky et al 2011]
- GR: $\gamma = 0.55$ and f(R): $\gamma = 0.41$. Signature for MGMs
- Enhanced power spectrum (EPS) in linear regime
- In particular, in order to fit the data one requires $n > 2$
- N-body sims. Chameleon mech tends to suppress EPS.

Extended scalar-tensor theories

- f(R) scalar-tensor theory with non-trivial dynamics
- General f(R): equivalent to $\mathscr{L} = \Phi R - U(\Phi), \quad \Phi \equiv df/d\varphi$
- No kinetic terms (KT)
- A mass term gives mass to the field: no propagating for large mass
- A massless field would be problematic, in general: 5th force
- What if a theory has no potential (e.g. shift symmetry)?
- What if we include general KT?

Galileon theory

[Nicolis, Rattazzi, Trincherini: PRD 2009]

- Inspired by DGP [Dvali et al: PLB 2000]
- Generalized scalar-tensor theory $\mathscr{L} \ni (\partial\varphi)^2 \nabla^2 \varphi / M^3$
- EOMs satisfy Galilean symmetry on Minkowski $\partial_\mu \varphi \rightarrow \partial_\mu \varphi + V_\mu$
- Model of brane-bending mode
- Needs non-linearities to make it not propagate for SSC
- Vainshtein mechanism

Horndeski theories

[Horndeski 1974]

- This is a generalization of Galileon
- Most general scalar-tensor theory with 2nd order EOMs
- It would model general DE induced by extra dimensions

$$\mathscr{L}=\sqrt{-g}\left[\frac{1}{2}M_P^2 R+P(\varphi,X)-G_3(\varphi,X)\nabla^2\varphi+\mathscr{L}_4+\mathscr{L}_5\right]$$

$$\mathscr{L}_4=G_4(\varphi,X)R+G_{4,X}[(\nabla^2\varphi)^2-(\nabla_\mu\nabla_\nu\varphi)(\nabla^\mu\nabla^\nu\varphi)]$$

$$\mathscr{L}_5=G_5(\varphi,X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\varphi)-\frac{1}{6}G_{5,X}[(\nabla^2\varphi)^3-3\nabla^2\varphi(\nabla_\mu\nabla_\nu\varphi)(\nabla^\mu\nabla^\nu\varphi)]$$

$$-\frac{1}{3}G_{5,X}(\nabla^\mu\nabla_\alpha\varphi)(\nabla^\alpha\nabla_\beta\varphi)(\nabla^\beta\nabla_\mu\varphi).\quad X\equiv-(\partial\varphi)^2/2$$

dRGT massive gravity

[de Rham, Gabadadze, Tolley: PRL 2011]

- Introducing 4 new scalar fields: Stuckelberg fields
- Then 4 sc dof, 4 vct dof, 2 Gws dof + 4 SF dof
- Unitary gauge (remove 4 SF dof): 4 sc , 4 vct, 2 Gws dof
- Constraints kill 2 sc dof and 2 vect dof: 2+2+2=6 dof
- dRGT kills one mode, the BD ghost. Finally only 5 dof.

No stable FLRW solutions

- FLRW background allowed [E. Gumrukcuoglu, C. Lin, S. Mukohyama: JCAP 2011]
- Late de Sitter solutions exist
- But no stable FLRW exists: one of the 5 dof is ghost
  [ADF, E. Gumrukcuoglu, S. Mukohyama: PRL 2012]
- Inhomogeneity? Anisotropies? [D'Amico et al: PRD 2011]
  [E. Gumrukcuoglu, C. Lin, S. Mukohyama: JCAP 2011. ADF, EG, SM: JCAP 2012]
- Something else? Introducing other fields: a quasidilaton scalar field
  [ADF, Mukohyama: PLB 2013][See also Huang, Piao, Zhou: PRD 2012]

Quasi-dilaton massive gravity

[D'Amico, Gabadadze, Hui, Pirtskhalava: PRD 2013]

- dRGT on FLRW: reduction of dof + ghost
- Avoid this behavior by introducing scalar field
- SF interacts with Stuckelberg fields/fiducial metric
- Non-trivial dynamics / perturbation behavior
- May heal the model? Still 2 GWs but massive: dof = 5 + 1

Symmetries of the model

- Lagrangian invariant under quasidilaton symmetry

$$\sigma \to \sigma_0, \quad \varphi^a \to e^{-\sigma_0/M_P}\varphi^a$$

- SFs satisfy Poincare symmetry

$$\varphi^a \to \varphi^a + c^a, \quad \varphi^a \to \Lambda^a{}_b \varphi^b$$

- Fiducial metric [ADF, Mukohyama: 2013]

$$\tilde{f}_{\mu\nu} = \eta_{ab}\partial_\mu\varphi^a\partial_\nu\varphi^b - \frac{\alpha_\sigma}{M_P^2 m_g^2} e^{-2\sigma/M_P}\partial_\mu\sigma\,\partial_\mu\sigma$$

Quasidilaton Lagrangian

- Following Lagrangian

$$\mathscr{L}=\frac{M_P^2}{2}\sqrt{-g}\left[R-2\Lambda-\frac{\omega}{M_P^2}\partial_\mu\sigma\,\partial_\nu\sigma+2m_g^2(\mathscr{L}_2+\alpha_3\mathscr{L}_3+\alpha_4\mathscr{L}_4)\right],$$

where

$$K^\mu{}_\nu=\delta^\mu{}_\nu-e^{\sigma/M_P}\left(\sqrt{g^{-1}\tilde{f}}\right)^\mu{}_\nu,$$

$$\mathscr{L}_2=\frac{1}{2}([K]^2-[K^2]),$$

$$\mathscr{L}_3=\frac{1}{6}([K]^3-3[K][K^2]+2[K^3]),$$

$$\mathscr{L}_4=\frac{1}{24}([K]^4-6[K]^2[K^2]+3[K^2]^2+8[K][K^3]-6[K^4]).$$

Background

- Give the ansatz

$$ds^2=-N^2dt^2+a^2d\vec{x}^2, \quad \phi^0=\phi^0(t), \quad \phi^i=x^i, \quad \sigma=\bar{\sigma}(t)$$

- Fiducial metric $\tilde{f}_{00}=-n(t)^2, \quad \tilde{f}_{ij}=\delta_{ij}$
- Defining $H=\dot{a}/(aN), \quad X=e^{\bar{\sigma}/M_P}/a, \quad r=an/N$
- de Sitter solution $\left(3-\frac{\omega}{2}\right)H^2=\Lambda+\Lambda_X, \quad \omega<6$

de Sitter solution

- Existence of de Sitter solution
- All expected 5 modes propagate
- Only if $\alpha_\sigma / m_g^2 > 0$ all the modes are well behaved: no ghost, and no classical instabilities.
- This same result can be generalized to general quasi-dilaton field.

Scalar contribution

- In the unitary gauge, integrating out auxiliary modes
- 2 scalar modes propagate: one with 0 speed, the other with speed equal to 1.
- Ghost conditions

$$0<\omega<6\,,\quad X^2<\frac{\alpha_\sigma H^2}{m_g^2}<r^2 X^2\,,\quad r>1$$

Vector and GW contributions

- Vector modes reduced action

$$\mathscr{L} = \frac{M_P^2}{16} a^3 N \left[\frac{T_V}{N^2} |\dot{E}_i^T|^2 - k^2 M_{GW}^2 |E_i^T|^2 \right], \quad T_V > 0$$

- Therefore

$$c_V^2 = \frac{M_{GW}^2}{H^2} \frac{r^2 - 1}{2\omega}, \quad M_{GW}^2 = \frac{(r-1) X^3 m_g^2}{X-1} + \frac{\omega H^2 (r X + r - 2)}{(X-1)(r-1)}, \quad M_{GW}^2 > 0$$

- GW reduces action

$$\mathscr{L} = \frac{M_P^2}{8} a^3 N \left[\frac{1}{N^2} |\dot{h}_{ij}^{TT}|^2 - \left(\frac{k^2}{a^2} + M_{GW}^2 \right) |h_{ij}^{TT}|^2 \right]$$

Status

- Self accelerating de Sitter solutions exist
- All 5+1 modes propagating
- All of them are stable
- Existence proof

Bigravity

[Hassan, Rosen: JHEP 2012]

- Promote fiducial metric to a dynamical component
- Introduce for it a new Ricci scalar
- Degrees of freedom in the 3+1 decomposition:
  - Total: (4 sc + 4 vt + 2GW) · 2
  - Gauge: 2 sc + 2 vt
  - Constraints: (2 sc + 2 vt) · 2 + 1 no-BD-ghost
  - Finally: T-G-C => 1 sc + 2 vt + 4 GW

Bimetric Lagrangian

- For the two metrics

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu , \quad d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu ,$$

- Introduce a ghost free action

$$\mathscr{L} = \sqrt{-g}\left[M_G^2 \left(\frac{R}{2} - m^2 \sum_{n=0}^{4} c_n V_n \left(Y^\mu{}_\nu \right) \right) + \mathscr{L}_m \right] + \frac{\kappa M_G^2}{2} \sqrt{-\tilde{g}}\, \tilde{R}$$

where

$$Y^\mu{}_\nu = \sqrt{g^{\mu\alpha} \tilde{g}_{\alpha\nu}} ,$$

$$[Y^n] = Tr\left(Y^n\right), \quad V_0 = 1, \quad V_1 = [Y],$$

$$V_2 = [Y]^2 - [Y^2], \quad V_3 = [Y]^3 - 3[Y][Y^2] + 2[Y^3],$$

$$V_4 = [Y]^4 - 6[Y]^2[Y^2] + 8[Y][Y^3] + 3[Y^2]^2 - 6[Y^4].$$

Background dynamics

- Assume FLRW ansatz

$$ds^2 = a^2(-dt^2 + d\vec{x}^2), \quad d\tilde{s}^2 = \tilde{a}^2(-\tilde{c}^2 dt^2 + d\vec{x}^2)$$

- Define $\xi = \tilde{a}/a, \quad H = \dot{a}/a^2$
- Existence of two branches [Comelli, Crisostomi, Pilo: JHEP 12]

$$\Gamma(\xi)(\tilde{c}\, a H - \dot{\tilde{a}}/\tilde{a}) = 0, \quad \Gamma = c_1 \xi + 4 c_2 \xi^2 + 6 c_3 \xi^3$$

- Physical branch: $\tilde{c} = \dot{\tilde{a}}/(\tilde{a}\, a H)$

GR-like dynamics

[ADF, Nakamura, Tanaka: PTEP 14]

- At low energies, Friedmann equation is recovered

$$3H^2 = \frac{\rho_m}{\tilde{M}_G^2}, \quad \xi \approx \xi_c, \quad \tilde{M}_G^2 = M_G^2 (1 + \kappa \xi_c^2), \quad \tilde{c} \approx 1,$$

where $\xi \to \xi_c$ when $\rho_m \to 0$

- The effective gravitational constant is different form the bare one, time independent
- Low energy cosmological dynamics consistent with data

Solar system constraints?

[ADF, Nakamura, Tanaka: PTEP 14]

- Gravitational potential of a star in he Minkowski limit
- Ansatz

$$ds^2 = -e^{u-v}dt^2 + e^{u+v}\left(dr^2 + r^2 d\Omega^2\right), \quad d\tilde{s}^2 = -\xi_c^2 e^{\tilde{u}-\tilde{v}}dt^2 + \xi_c^2 e^{\tilde{u}+\tilde{v}}\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right).$$

- Defining $\tilde{r} = e^{R(r)}r\,, \quad C = \dfrac{d\ln\Gamma}{d\ln\xi}\,, \quad C \gg 1\,,$
- Then at second order, u → 0, (Vainshtein mechanism) with same effective gravitational constant $\nabla^2 v \approx -\dfrac{\rho_m}{\tilde{M}_G^2}$

[Babichev, Deffayet, Esposito-Farese, PRL 11; Kimura, Kobayashi, Yamamoto PRD 12]

Graviton oscillations

[ADF, Nakamura, Tanaka: PTEP 14]

- Study propagation of 4 GW – coupled 2 by 2

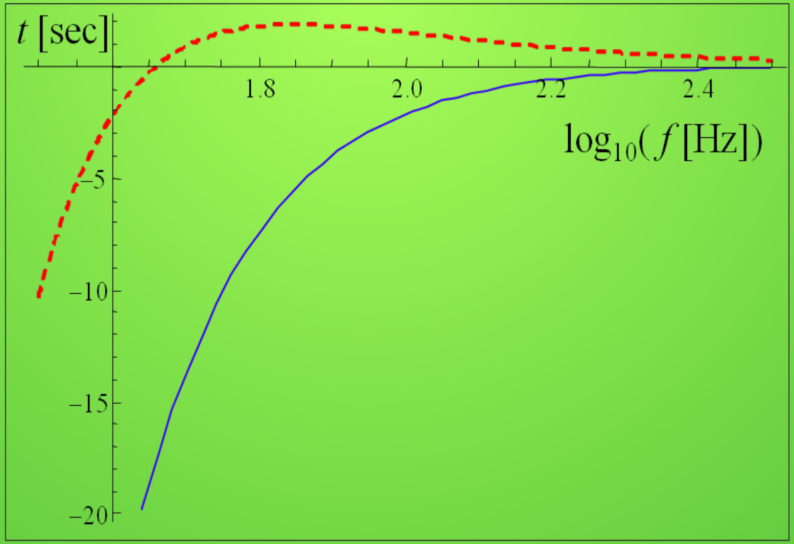
$$\ddot{h} - \nabla^2 h + m^2 \Gamma_c (h - \tilde{h}) = 0,$$

$$\ddot{\tilde{h}} - \tilde{c}^2 \nabla^2 \tilde{h} + \frac{m^2 \Gamma_c}{\kappa \xi_c^2} (h - \tilde{h}) = 0.$$

- Define $\mu^2 = \frac{(1 + \kappa \xi_c^2) \Gamma_c m^2}{\kappa \xi_c^2}$
- Eingenmodes: one massless and one massive μ
- Graviton oscillations possible

Inverse chirp signal

- For NS–NS: $h = A(f) e^{i\Phi(f)} [B_1 e^{i\delta\Phi_1(f)} + B_2 e^{i\delta\Phi_2(f)}]$
- Graviton modes have inverse chirp signal:
  arrival time vs frequency reversed for the second (red) mode



Constraints

- Cosmological dynamics similar to GR
- To pass solar system tests: need hierarchy in the graviton mass term
- Weak field approximations + 2nd order perturbations

$$r_V = O\left(\left(C\, r_g \lambda_\mu^2\right)^{1/3}\right), \quad \nabla^2 v = -\tilde{M}_G^{-2} \rho_m$$

- Black holes? [Babichev, Fabbri CQG 13, 1401.6871]

Conclusions

- Dark Energy/Gravity: active field of research
- What is gravity?
- Yet, a field to investigate
- Can the graviton (or one of them) be massive?
- Experimental and theoretical research is needed